

[This question paper contains 8 printed pages.] 20/12/24

Your Roll No.....

Sr. No. of Question Paper : 1064

I

Unique Paper Code : 2352013501

Name of the Paper : Metric Spaces

Name of the Course : **B.Sc. (Hons.) Mathematics
(DSC-13)**

Semester : V

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. Attempt all question by selecting two parts from each question.
3. Part of the questions to be attempted together.
4. All questions carry equal marks.
5. Use of Calculator not allowed.

P.T.O.

1. (a) Let (X, d) be a metric space. Define the function

$$d': X \times X \rightarrow \mathbf{R} \text{ by } d' = \frac{|x-y|}{1+|x-y|} \text{ Show that } d' \text{ is a metric on } X. \text{ Besides, } d'(x, y) < 1 \text{ for all } x, y \in X. \quad (7.5)$$

- (b) Let $X = C[a, b]$ be the space of all continuous

$$\text{functions on } [a, b]. \text{ Define } d(x, y) = \int_a^b |f(x) -$$

$g(x)| dx$, then check whether this metric imply

pointwise Convergence or not. (7.5)

- (c) Define Cauchy Sequence and Complete metric space. Let X be any non-empty set and d be defined by

$$d(x, y) = \begin{cases} 0, & x = y \\ 1, & x \neq y \end{cases}$$

then show that (X, d) is a Complete metric space.
(7.5)

2. (a) Let (X, d) be a metric space. Then show that

(i) $\emptyset$ and X are open sets in (X, d) ;

(ii) the union of an arbitrary family of open sets is open;

(iii) the intersection of any finite family of open sets is open.
(7.5)

(b) Let A be a subset of a metric space (X, d) . Then prove that

(i) A° is the largest open subset of A . (3.5)

(ii) A is open if and only if $A = A^\circ$. (4)

(c) (i) Let (X, d) be a metric space and $F \subseteq X$.
Then show that a point x_0 is a limit point of F if and only if it is possible to select from the set F a sequence of distinct

$x_1, x_2, \dots, x_n, \dots$ such that $\lim_{n \rightarrow \infty} d(x_n, x_o) = 0$.

(4.5)

(ii) Let $A \subseteq [0, 1]$ and $F = \{f \in C[0, 1] :$

$f(t) = 0, \forall t \in A\}$. Show that F is a closed subset of $C[0, 1]$ equipped with the uniform metric.

(3)

3. (a) Let (X, d) be a metric space and $F \subseteq X$. Then show that the following statements are equivalent:

(i) $x \in \bar{F}$;

(ii) $S(x, \varepsilon) \cap F \neq \emptyset$ for every open ball $S(x, \varepsilon)$ centred at x ;

(iii) There exists an infinite sequence $\{x_n\}$ of points (not necessarily distinct) of F such that

$$\lim_{n \rightarrow \infty} x_n = x. \quad (7.5)$$

(b) State and prove Cantor's intersection theorem.

(7.5)

(c) (i) If Y is a nonempty subset of a metric space (X, d) , and (Y, d_y) is complete, then show that Y is a closed in X . (3.5)

(ii) Let (X, d) be a complete metric space and Y a closed subset of X . Then show that (Y, d_y) is a complete space. (4)

4. (a) Prove that a mapping $f: X \rightarrow Y$ is continuous on X if and only if $f^{-1}(G)$ is open in X for all open subsets G of Y . (7.5)

(b) Let $T: X \rightarrow X$ be a contraction mapping of the complete metric space (X, d) . Then show that T has a unique fixed point. (7.5)

(c) Show that the metric spaces (X, d) and (X, ρ)

$$\text{where } \rho(x, y) = \frac{d(x, y)}{1 + d(x, y)} \text{ are equivalent. (7.5)}$$

5. (a) Let (X, d) be a metric space. Then show that the following statements are equivalent:

(i) (X, d) is disconnected;

(ii) there exists a continuous mapping of (X, d) onto the discrete two element space (X_o, d_o) .
(7.5)

(b) Let $I = [1, 1]$ and let $f: I \rightarrow I$ be continuous. Then show that there exists a point $c \in I$ such that $f(c) = c$. Discuss the result if $I = [-1, 1]$. Discuss the result if $I = [-1, 1)$ and $I = [-1, \infty)$. (4+3.5)

(c) (i) If C is a connected subset of a disconnected metric space $X = A \cup B$, where A, B are

nonempty and $\bar{A} \cap B = \emptyset = A \cap \bar{B}$, then show that either $C \subseteq A$ or $C \subseteq B$.

- (ii) If Y is a connected set in a metric space (X, d) then show that any set Z such that is $Y \subseteq Z \subseteq \bar{Y}$ connected. (4+3.5)

6. (a) Let (X, d) be a metric space. Then show that the following statements are equivalent:

(i) every infinite set in (X, d) has at least one limit point in X ;

(ii) every infinite sequence in (X, d) contains a convergent subsequence. (7.5)

(b) If f is a one-to-one continuous mapping of a compact metric space (X, d_x) onto a metric space (Y, d_y) , then show that f^{-1} is continuous on Y and, hence, f is a homeomorphism of (X, d_x) onto (Y, d_y) . (7.5)

- (c) Let A be a compact subset of a metric space (X, d) . Show that for any $B \subseteq X$, there is a point $p \in A$ such that $d(p, B) = d(A, B)$.. (7.5)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1102

I

Unique Paper Code : 2352013502

Name of the Paper : Ring Theory

Name of the Course : B.Sc. (H) Mathematics

Semester : V – DSC-14

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory and are of 15 marks each.
3. Attempt any six parts from Question 1. Each part is of 2.5 marks.
4. Attempt any two parts from each of the Questions 2 to 6. Each part is of 7.5 marks.
5. Use of calculator is not allowed.

P.T.O.

1.
 - (i) Find all the units of $\mathbb{Z}_7[x]$.
 - (ii) Check whether $\mathbb{Q} \oplus \mathbb{Q}$ is an integral domain or not.
 - (iii) Give an example of a subring S of a ring R which is not an ideal of R .
 - (iv) Prove that a ring homomorphism carries an idempotent to an idempotent.
 - (v) Let ϕ be a ring homomorphism from a ring R to a ring S . If R has unity 1 , $S \neq \{0\}$ and ϕ is onto then prove that $\phi(1)$ is the unity of S .
 - (vi) Let $f(x) = 2x^5 + 14x^2 - 21x + 7$. Is $f(x)$ an irreducible polynomial over $\mathbb{Q}$? Justify your answer.
 - (vii) Let D be an integral domain. Suppose that $p, q \in D$ and $q \neq 0$. Show that if p is not a unit, then $\langle pq \rangle$ is a proper subset of $\langle q \rangle$.
 - (viii) Explain why $3x^2 + 6$ is reducible over $\mathbb{Z}$.
2.
 - (a) Prove that intersection of two subrings in a ring R is a subring of R . Is the union of two subrings necessarily a subring of R ? Justify your answer.
 - (b) Find all the units, zero divisors and idempotent elements in $\mathbb{Z}_3 \oplus \mathbb{Z}_6$.

- (c) Prove that $\mathbb{Z}_n$, the ring of integers modulo n , is a field if and only if n is a prime.
3. (a) Let R be a commutative ring with unity and let $U(R)$ denote the set of units of R . Prove that $U(R)$ is a group under multiplication. Also, find $U(\mathbb{Z}[i])$.
- (b) Define the characteristic of a ring. Prove that the characteristic of an integral domain is either 0 or prime.
- (c) Prove that in a commutative ring R with unity, an ideal A is a maximal ideal if and only if $\frac{R}{A}$ is a field.
4. (a) Prove that the ideal $\langle x \rangle$ is a prime ideal in $\mathbb{Z}[x]$ but not a maximal ideal in $\mathbb{Z}[x]$.
- (b) Let ϕ be a ring homomorphism from a ring R onto a ring S . Prove that $\frac{R}{\text{Ker } \phi} \approx S$.
- (c) Determine all ring homomorphisms from $\mathbb{Z}_4 \rightarrow \mathbb{Z}_{10}$.
5. (a) Let F be a field and let $I = \{a_0 + a_1x + \dots + a_nx^n : a_0, a_1, \dots, a_n \in F \text{ and } a_0 + a_1 + \dots + a_n = 0\}$. Show that I is an ideal of $F[x]$ and find a generator for I .

(b) Let $f(x) = 5x^4 + 3x^3 + 1$ and $g(x) = 3x^2 + 2x + 1 \in \mathbb{Z}_7[x]$. Determine the quotient and remainder obtained when $f(x)$ is divided by $g(x)$.

(c) Prove that the product of two primitive polynomials is a primitive polynomial.

6. (a) Show that $p(x) = x^3 + x + 1$ is an irreducible polynomial over $\mathbb{Z}_2$.

Let $M = \langle x^3 + x + 1 \rangle$ be an ideal of $\mathbb{Z}_2[x]$.

Show that $F = \frac{\mathbb{Z}_2[x]}{M}$ is a field of order 8. Exhibit

all the 8 elements of F . Find the product of $x^2 + x + 1 + M$ and $x^2 + 1 + M$ and express it as a member of F .

(b) In a principal ideal domain, prove that the element is irreducible if and only if it is prime.

(c) Show that integral domain $\mathbb{Z}[t]$ is Euclidean Domain. Is $\mathbb{Z}[i]$ a Unique Factorization Domain? Justify.

This question paper contains 4 printed pages]

24/12/24

Roll No.

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S. No. of Question Paper : 1121

Unique Paper Code : 2352013503

Name of the Paper : Partial Differential Equations

Name of the Course : Bachelor of Science (Honours Course) Mathematics

Semester : V

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the six questions are compulsory.

Attempt any two parts from each question.

Use of calculator is not allowed.

1. (a) Define the linear and non-linear partial differential equations with examples. Find the first order partial differential equation satisfied by the family of right circular cones whose axes coincide with the z-axis, and is given by :

$$x^2 + y^2 = (z - c)^2 \tan^2 \alpha.$$

- (b) Solve the following initial value problem using method of characteristics :

$$u_t + 2uu_x = v - x, \quad v_t - cv_x = 0 \quad \text{with} \quad u(x, 0) = x, \quad v(x, 0) = x.$$

- (c) Solve the equation $u_x + xu_y = y$ with the Cauchy data $u(1, y) = 2y$.

P.T.O.

2. (a) Reduce the following equation into canonical form and then find the general solution :

$$u_x + 2xyu_y = x.$$

- (b) Apply the method of separation of variables $u(x, y) = f(x)g(y)$ to solve the following equation :

$$u_x + 2u_y = 0, \quad u(0, y) = 3e^{-2y}.$$

- (c) Find a complete integral of the equation by using Charpit's method :

$$p = (z + qy)^2.$$

3. (a) Show that the equation of motion of the vibrating string is :

$$u_{tt} = c^2 u_{xx}, \quad c^2 = T/\rho,$$

where T is the tension at the end point of the string and ρ is the density.

- (b) Classify and determine the region in which the following equation is hyperbolic, parabolic, or elliptic, and transform the equation in the respective region to canonical form :

$$x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0.$$

- (c) Find the traffic density $\rho(x, t)$, satisfying :

$$\frac{\partial \rho}{\partial t} + x \sin(t) \frac{\partial \rho}{\partial x} = 0,$$

with the initial condition $\rho_0(x) = 1 + \frac{1}{1+x^2}$.

4. (a) Transform the following equation to the form $u_{\xi\eta} = cv$, $c = \text{constant}$,

$$3u_{xx} + 7u_{xy} + 2u_{yy} + u_y + u = 0,$$

by introducing the new variables $v = ue^{a\xi + b\eta}$, where a and b are undetermined coefficients.

- (b) Define the homogeneous and non-homogeneous of partial differential equations with examples, and find the general solution of the equation :

$$u_{xxxx} - u_{yyyy} = 0.$$

- (c) Find the general solution of the equation :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y + e^{2x+y}.$$

5. (a) Determine the solution of the initial-value problem for the semi-infinite string with a fixed end :

$$\begin{aligned} u_{tt} &= c^2 u_{xx}, & 0 < x < \infty, t > 0 \\ u(x, 0) &= f(x), u_t(x, 0) = g(x), & 0 \leq x < \infty \\ u(0, t) &= 0, & 0 \leq t < \infty \end{aligned}$$

- (b) Determine the solution of the initial-value problem for the semi-infinite string with free end :

$$\begin{aligned} u_{tt} &= 9u_{xx}, & 0 < x < \infty, t > 0 \\ u(x, 0) &= 0, u_t(x, 0) = x^3, & 0 \leq x < \infty \\ u_x(0, t) &= 0, & 0 \leq t < \infty \end{aligned}$$

- (c) Determine the solution of the initial-value problem with non-homogeneous boundary conditions :

$$\begin{aligned} u_{tt} &= c^2 u_{xx}, & 0 < x < \infty, t > 0 \\ u(x, 0) &= \sin x, u_t(x, 0) = x^2, & 0 \leq x < \infty \\ u(0, t) &= x, & 0 \leq t < \infty \end{aligned}$$

6. (a) Determine the solution of the Cauchy problem for non-homogeneous wave equation :

$$u_{tt} - c^2 u_{xx} = \sin x = 0, \quad u(x, 0) = \cos x, \quad u_t(x, 0) = 1 + x.$$

- (b) Determine the solution of the initial-value problem :

$$u_{tt} = 16u_{xx}, \quad 0 < x < \infty, t > 0$$

$$u(x, 0) = 1 + x, \quad u_t(x, 0) = x^3, \quad 0 \leq x < \infty$$

$$u_x(0, t) = \cos x, \quad 0 \leq t < \infty$$

- (c) Determine the solution of the initial-value problem :

$$u_{tt} - c^2 u_{xx} = 0 \quad 0 < x < \infty, t > 0$$

$$u(x, 0) = \log(1 + x^2), \quad u_t(x, 0) = 2, \quad 0 \leq x < \infty$$

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1227

I

Unique Paper Code : 2353010007

Name of the Paper : DSE-3(i) : Mathematical Data
Science

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V, DSE

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **three** parts from **Q1-Q4.** and two parts from **Q5.-Q6.**
3. Parts of the questions to be attempted together.
4. **All** questions carry equal marks. **All** parts of a question carry equal marks. Marks are indicated.
5. Use of non-programmable Scientific Calculator is allowed.

P.T.O.

1. (a) Differentiate between structured and unstructured data. (5)

(b) What is preprocessing of data. Explain with the help of an example. (5)

(c) Consider the data of coffee shops with following attributes: Name of the coffee shop, Revenue in Rs., Pin code of the shop, Average monthly customers. Briefly explain the attributes as quantitative and qualitative. (5)

(d) Define discrete quantitative and continuous quantitative data. Give an example of each. (5)

2. (a) Write five steps of data science and explain these briefly. (5)

(b) Explain any two tabular data formats out of CSV, Excel, Data Frame, XML or similar. (5)

(c) What are anomalies in data science? Explain anomalies with examples. (5)

(d) Give an overview of data source format SQL or HDF5. (5)

3. (a) State Chebyshev Inequality. Consider a probability density function f and a random variable $X \sim f$. If $E[X] = 20$ and $\text{Var}[X] = 9$, find the value of $\Pr[X \geq 50]$. (5)

(b) Explain the term probably approximately correct with an example. Consider sets $A = \{1,2,4,8\}$ and $B = \{1,2,3\}$. Calculate the Jaccard distance between A and B . (5)

(c) Define L_p distances mathematically and sketch these geometrically for $p = 1, 2$ and ∞ . (5)

(d) Consider two vectors $a = (1, 2, -4, 3, -6)$ and $b = (1, 2, 5, -2, 1)$ in $\mathbb{R}^5$. Find Kullback-Liebler divergence. (5)

4. (a) Find the regression line for the given data

height (in): 66 68 60 70 65 61 74 73 75 67

weight (lbs): 160 170 110 178 155 120 223 215 235 164

(5)

(b) Explain polynomial regression, Consider the input data set with $n = 3$ points $\{(2, 1), (3, 6), (4, 5)\}$.

Find polynomial expansion of x generates with $p = 5$. (5)

(c) Define the Gradient in data sciences and find the value of the Gradient for

$$f(x,y,z) = 3x^2 - 2y^3 - 2xe^z \text{ at } (3,-2,1). \quad (5)$$

(d) Consider two-variable function $f = (x-5)^2 + (y+2)^2 - 2xy$. Starting with $(x, y) = (0,2)$, using the gradient descent algorithm for the function, perform 3 iterations and report the function value at the end of each step. (5)

5. (a) Find Singular Value Decomposition (SVD) of the

$$\text{matrix } A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}. \quad (7.5)$$

- (b) Use Lloyd's Algorithm for k-means clustering to divide the following data into two clusters with initial cluster centers $C_1 = (2,1)$ and $C_2 = (2,3)$

x	1	2	2	3	4	5
y	1	1	3	2	3	5

(7.5)

- (c) Reduce the feature 2-dimensional data of the following into one dimensional using Principal Component Analysis (PCA)

Class →	C_1	C_2	C_3	C_4
Feature ↓				
X_1	4	8	13	7
X_2	11	4	5	14

(7.5)

6. (a) Consider a matrix $A \in \mathbb{R}^{8 \times 4}$ with squared singular values $\sigma_1^2 = 10$, $\sigma_2^2 = 5$, $\sigma_3^2 = 2$ and $\sigma_4^2 = 1$.

(i) Find the rank of matrix A .

(ii) Find the value of $\|A - A_2\|_F^2$, where A_2 is the best rank-2 approximation of A . (7.5)

(b) Draw Voronoi diagram with three points $A(-6,7)$, $B(-6,-3)$ and $C(2,5)$. (7.5)

(c) Consider a set of 1 - dimensional data points

$$(x_1 = 0, y_1 = +1), (x_2 = 1, y_1 = -1), (x_3 = 2, y_1 = +1),$$

$$(x_4 = 4, y_1 = +1), (x_5 = 6, y_1 = -1), (x_6 = 7, y_1 = -1),$$

$$(x_7 = 8, y_1 = +1), (x_8 = 9, y_1 = -1).$$

Predict -1 or $+1$ using a kNN (k-nearest neighbor) classifier with $k = 3$ on $x = 3$ and $x = -1$.

(7.5)

[This question paper contains 8 printed pages.]

01/01/2025

Your Roll No.....

Sr. No. of Question Paper : 1228

I

Unique Paper Code : 2353010008

Name of the Paper : Linear Programming and Applications (DSE)

Name of the Course : B.Sc. (H) Mathematics

Semester : V

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.

1. (a) Solve the following linear programming problem by Graphical Method:

$$\begin{aligned} \text{Minimize} \quad & z = x_1 + x_2 \\ \text{subject to} \quad & x_1 + x_2 \geq 2 \\ & 5x_1 + 9x_2 \leq 45 \\ & x_2 \leq 4 \\ & x_1 \geq 0, x_2 \geq 0. \end{aligned}$$

P.T.O.

(b) Examine the convexity of the set $S = \{(x_1, x_2): x_2^2 \geq 4x_1\}$. Justify your conclusion. Further prove or disprove that intersection of two convex sets is a convex set.

(c) Solve the following problem by simplex method:

$$\begin{array}{ll}
 \text{Minimize} & x_1 + 2x_2 - 4x_3 \\
 \text{subject to} & x_1 + x_2 + 2x_3 \leq 9 \\
 & x_1 + x_2 - x_3 \leq 2 \\
 & -x_1 + x_2 + x_3 \leq 4 \\
 & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.
 \end{array}$$

2. (a) Consider the following problem:

$$\begin{array}{ll}
 \text{Minimize} & z = cx \\
 \text{subject to} & Ax \geq b, \\
 & x \geq 0,
 \end{array}$$

Let $(x_B, 0)$ be a basic feasible solution corresponding to basis B such that $z_j - c_j \leq 0$ for all non-basic variables x_j . Prove that $(x_B, 0)$ is an optimal basic feasible solution.

- (b) Find all the basic feasible solutions for the following system of equations:

$$\begin{aligned}x_1 + 2x_2 + x_3 &= 4 \\ 2x_1 + x_2 + 5x_3 &= 5.\end{aligned}$$

Classify the obtained solutions as degenerate or non-degenerate.

- (c) Reduce the feasible solution, $x_1 = 2, x_2 = 3, x_3 = 1$ of the following system to two different basic feasible solutions:

$$\begin{aligned}2x_1 + x_2 + 4x_3 &= 11 \\ 3x_1 + x_2 + 5x_3 &= 14.\end{aligned}$$

3. (a) Solve the following problem by Big-M method

$$\text{Maximize } z = 6x_1 + 4x_2$$

$$\begin{aligned}\text{subject to } & 2x_1 + 3x_2 \leq 30 \\ & 3x_1 + 2x_2 \leq 24 \\ & x_1 + x_2 \geq 3 \\ & x_1, x_2 \geq 0\end{aligned}$$

Is the solution unique? If not, give two different solutions.

(b) Solve the following problem by Two-phase method

$$\begin{aligned} \text{Minimize } z &= -5x_1 + 4x_2 - 3x_3 \\ \text{subject to } 2x_1 + x_2 - 6x_3 &= 20 \\ 6x_1 + 5x_2 + 10x_3 &\leq 76 \\ 8x_1 - 3x_2 + 6x_3 &\leq 50 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

(c) Solve the following system of simultaneous equations using simplex method:

$$\begin{aligned} x_1 + x_2 &= 1 \\ 2x_1 + x_2 &= 3 \end{aligned}$$

4 (a) (i) Prove that if either the primal or the dual problem has an unbounded objective function

value, then the other problem has no feasible solution.

- (ii) Show that the dual of the dual is primal for the following primal problem

$$\begin{aligned} \text{Minimize } z &= x_1 + 2x_2 - 3x_3 \\ \text{subject to } 4x_1 + 5x_2 - 6x_3 &= 7 \\ 8x_1 - 9x_2 + 10x_3 &\leq 11 \\ x_1, x_2 &\geq 0, x_3 \text{ unrestricted.} \end{aligned}$$

- (b) Obtain the dual of the following primal problem:

$$\begin{aligned} \text{Minimize } z &= 3x_1 - 2x_2 + 4x_3 \\ \text{subject to } 3x_1 + 5x_2 + 4x_3 &\geq 7 \\ 6x_1 + x_2 + 3x_3 &\geq 4 \\ 7x_1 - 2x_2 - x_3 &\leq 10 \\ x_1 - 2x_2 + 5x_3 &\geq 3 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

- (c) Solve the following problem using Complementary Slackness theorem

$$\begin{aligned}
 &\text{Minimize } z = 2x_1 + 9x_2 + x_3 \\
 &\text{subject to } x_1 + 4x_2 + 2x_3 \geq 5 \\
 &\quad \quad \quad 3x_1 + x_2 + 2x_3 \geq 4 \\
 &\quad \quad \quad x_1, x_2, x_3 \geq 0.
 \end{aligned}$$

5. (a) For the following cost minimization Transportation Problem, find the initial basic feasible solution by using North West Corner Rule, Least Cost Method and Vogel's Approximation Method. Compare the three solutions in terms of cost:

Machine	I	II	III	IV	V	Supply
A	2	11	10	3	7	4
B	1	4	7	2	1	8
C	3	9	4	8	12	9
Demand	3	3	4	5	6	

- (b) Solve the following cost minimization Transportation Problem:

	I	II	III	IV	Availability
A	19	30	50	10	7
B	70	30	40	60	9
C	40	8	70	20	18
Requirements	5	8	7	14	

(c) Find the optimal solution of the Assignment Problem with the following cost matrix :

	U	V	X	Y	Z
I	2	9	2	7	1
II	6	8	7	6	1
III	4	6	5	3	1
IV	4	2	7	3	1
V	5	3	9	5	1

6. (a) (i) Define Saddle Point and Mixed Strategy for a "Two-Person Zero Sum" game.

(ii) Use maxmin and minmax Principle to find the saddle point, if it exists, for the following pay-off matrix

$$\begin{bmatrix} 3 & -1 & 5 \\ 6 & 4 & 0 \\ 10 & 8 & 6 \end{bmatrix}$$

- (b) Convert the following game problem into a pair of linear programming problems for players A and B and provide the optimal strategies of the two players and value of the game.

Player B

$$\text{Player A} \quad \begin{bmatrix} 2 & 3 & 0 & -2 \\ 1 & 2 & 3 & 2 \end{bmatrix}$$

- (c) Find the inverse of the matrix using Simplex Method:

$$\begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$$

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16/12/24

Your Roll No.....

Sr. No. of Question Paper : 5832

I

Unique Paper Code : 32351501

Name of the Paper : BMATH511 – Metric Spaces

Name of the Course : B.Sc. (Hons.) Mathematics
(LOCF)

Semester : V, Core

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any two parts from each question.
4. In the question paper, given notations have their usual meaning unless and until stated otherwise.

P.T.O.

1. (a) (i) Define a metric space. Show that the non-negativity metric axiom is implied by the other metric axioms of coincidence, symmetry and triangle inequality.

(ii) Let X be a non-empty set.

Define for $x, y \in X$

$$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ \sqrt{2} & \text{if } x \neq y \end{cases}$$

Prove that d is a metric on X . (3+3)

- (b) (i) Find the open ball in $C_R[0, 1]$ centred at $f(t) = t^2$ of radius 1. Give its geometrical interpretation.

(ii) Show that $(3, 5)$ is an open ball in $\mathbb{R}$. Find its centre and radius. (4+2)

- (c) Suppose $X = C_{\mathbb{R}}[a, b]$, the set of all continuous functions from $[a, b]$ to $\mathbb{R}$ and $d(f, g) = \sup \{|f(x) - g(x)| : x \in [a, b]; a < b\}$ be the associated metric. Then prove that (X, d) is a complete metric space.
- (6)

2. (a) (i) Suppose (X, d) is a metric space and F is a finite subset of X . Prove that the derived set F' of F is empty.

- (ii) Let A be a finite subset of $\mathbb{R}$ with usual metric. Is A open in $\mathbb{R}$? Justify your answer. If not, then give an example of a metric space in which a finite set may be open.

(2.5+4)

- (b) (i) Consider the subspace $X = I \cup J$ of $\mathbb{R}$, where $I =]0, 1[$ and $J = [4, 7[$. Show that $d(\sup_x I, J) > 0$.

- (ii) Find the interior and derived sets of Z and Q in $\mathbb{R}$ with usual metric. Determine which one of these sets is open or closed or none in $\mathbb{R}$. Justify your answer.
- (2.5+4)

- (c) (i) Is $Q \cup \{\sqrt{2}\}$ dense in $\mathbb{R}$ with usual metric?

Justify your answer.

- (ii) Give an example of a set which is open in a subspace Y of a metric space (X, d) but not open in X . Under what condition every open set in a subspace Y of a metric space (X, d) is open in X ? Justify your answer.

(2+4.5)

3. (a) Prove that a mapping $f: X \rightarrow Y$ is continuous on X if and only if $f^{-1}(G)$ is open in X for all open subsets G of Y . (6)

- (b) Define a uniformly continuous function on a metric space. Prove that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2$ is not uniformly continuous, whereas the function $f: [0, 1] \rightarrow \mathbb{R}$ defined by $f(x) = x^2$ is uniformly continuous. (6)

- (c) Let f be an isometry between metric spaces (X_1, d_1) and (X_2, d_2) . Show that f is a homeomorphism between X_1 and X_2 . Is the converse true? Justify your answer. (6)

4. (a) Show that the metrics d_1 , d_2 and d_∞ defined on $\mathbb{R}^n$ by

$$d_1(x, y) = \sum_{i=1}^n |x_i - y_i|;$$

$$d_2(x, y) = \left(\sum_{i=1}^n |x_i - y_i|^2 \right)^{\frac{1}{2}}; \text{ and}$$

$$d_\infty(x, y) = \max \{|x_i - y_i| : 1 \leq i \leq n\}$$

are equivalent. (6.5)

- (b) Let $X = [0, 2\pi] \subset \mathbb{R}$ and $Y = \{z \in \mathbb{C} : |z| = 1\}$, the unit circle in the complex plane. Consider the map $f : X \rightarrow Y$ defined by $f(t) = e^{it}$, $0 \leq t \leq 2\pi$. Show that the function f is one-to-one, onto, continuous but not a homeomorphism. (6.5)

(c) Let T be a continuous mapping of a complete metric space into itself. Define T^n inductively by $T^1 = T$ and $T^{n+1} = T \circ T^n$. If T^k is a contraction for some positive integer k , then the equation $Tx = x$ has one and only one solution. Moreover, for any $x \in X$, the sequence $\{T^n x\}_{n \geq 1}$ converges to that solution. (6.5)

5. (a) Let (X, d) be a metric space. Then show that the following statements are equivalent:

- (i) (X, d) is disconnected;
- (ii) There exist two nonempty disjoint subsets A and B , both open in X , such that $X = A \cup B$;
- (iii) There exist two nonempty disjoint subsets A and B , both closed in X , such that $X = A \cup B$.
- (iv) There exists a proper subset of X that is both open and closed in X . (6)

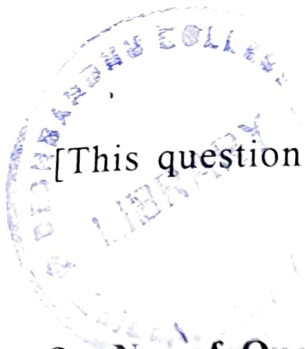
(b) Let (X, d_x) be a connected metric space and $f: (X, d_x) \rightarrow (Y, d_y)$ be a continuous mapping. Then show that the space $f(X)$ with the metric induced from Y is connected. (6)

(c) Prove that the intervals are connected sets in $(\mathbb{R}, d)$, the metric space of real numbers with the usual metric. (6)

6. (a) Let (X, d) be a metric space. Then prove that the following statements are equivalent:

- (i) (X, d) is compact;
- (ii) every collection of closed sets in (X, d) with empty intersection has a finite subcollection with empty intersection;
- (iii) every collection of closed sets in (X, d) with the finite intersection property has nonempty intersection. (6.5)

- (b) If f is a one-to-one continuous mapping of a compact metric space (X, d_X) onto a metric space (Y, d_Y) , then show that f^{-1} is continuous on Y and hence, f is a homeomorphism of (X, d_X) onto (Y, d_Y) . (6.5)
- (c) If the real-valued function f is continuous on the closed bounded interval $[a, b]$, then prove that f is uniformly continuous on $[a, b]$. (6.5)



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[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5886

I

Unique Paper Code : 32351502

Name of the Paper : Group Theory-2 (Core)

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Question No.1 has been divided in 10 parts and each part is of 1.5 marks.
4. Each question from 2 to 6 has 3 parts and each part is of 6 marks.

P.T.O.

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5. Attempt **any two** parts from each question.

1. State True (T) or False (F). Justify your answer in brief.

(a) $\text{Aut}(\mathbb{Z}_n)$ is always Abelian group, where $\mathbb{Z}_n = \{0, 1, 2, \dots, n-1\}$ is group under addition modulo n .

(b) $G/Z(G)$ is isomorphic to $\text{Aut}(G)$, where $Z(G)$ is Centre of group, where $\text{Aut}(G)$ is group of automorphism.

(c) Group of order 9 is isomorphic to $\mathbb{Z}_9$ or $\mathbb{Z}_3 \oplus \mathbb{Z}_3$, where $\mathbb{Z}_m \oplus \mathbb{Z}_n$ is an external direct product of group $\mathbb{Z}_m$ and $\mathbb{Z}_n$.

- (d) $\mathbb{Z}_2 \oplus \mathbb{Z}_5$ is isomorphic to $\mathbb{Z}_{10}$.
- (e) Order of an element $(1, 3)$ is 6 in $\mathbb{Z}_3 \oplus \mathbb{Z}_6$.
- (f) Kernel of a trivial action of a group G on a set A is $\{e\}$, where e is the identity of G .
- (g) $8 = 1+1+3 +3$ is the class equation of group of order 8.
- (h) Let G be a finite group and p be a prime that divides order of G . Then G has an element of order p .
- (i) Group of order 40 has a proper normal subgroup.

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(j) Group of order 35 is always cyclic.

2. (a) Let G be a group. Prove that the mapping $f(g) = g^{-1} \forall g \in G$ is an automorphism if and only if G is Abelian.

(b) Prove that for every positive integer n , $\text{Aut}(\mathbb{Z}_n)$ is isomorphic to $U(n)$.

(c) Define the commutator subgroup G' of a group G . Prove that G' is a normal subgroup of G and G/G' is Abelian.

3. (a) Find the number of cyclic subgroups of order 10 in the external direct product $\mathbb{Z}_{100} \oplus \mathbb{Z}_{25}$.

- (b) Prove that if a group G is the internal direct product of a finite number of subgroups $H_1, H_2, \dots, H_n$ then G is isomorphic to the external direct product of $H_1, H_2, \dots, H_n$
- (c) Prove that every group of order p^2 , where p is prime, is isomorphic to $\mathbb{Z}_{p^2}$ or $\mathbb{Z}_p \oplus \mathbb{Z}_p$.
4. (a) Let G be an abelian group of order 120 and G has exactly three elements of order 2. Determine the isomorphism class of G .
- (b) Let G be a group and A be any subset of G . Define the centralizer $C_G(A)$ of A in G . Further for dihedral group D_8 of order 8 with usual

generators r and s . Let $A = \{1, r, r^2, r^3\}$, subgroup of D_8 , then show that $C_{D_8}(A) = A$.

- (c) Let G be a group acting on a nonempty set A , then show that a relation on A defined by $a \sim b$ if and only if $a = g.b$ for some $g \in G$, is an equivalence relation. Further show that, for each $a \in A$, the number of elements in the equivalence class containing a is $|G : G_a|$, the index of the stabilizer of a .

5. (a) Find all conjugate classes of the Quaternion group Q_8 . Also, Verify its Class Equation.

- (b) Show that two elements are conjugate in symmetric group S_n if and only if they have the same cycle type.
- (c) Find a representative for each of conjugacy class of elements of order 4 in S_{12} .
6. (a) Show that a group of order pq , where p and q are distinct primes with $p < q$, $p \nmid (q-1)$, is cyclic.
- (b) If G is a group of order 30, show that either sylow 3-subgroup or sylow 5-subgroup is normal in G .

(c) State Index Theorem. Hence or otherwise, prove that a group of order 12 is not Simple.

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[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5969

I

Unique Paper Code : 32357501

Name of the Paper : Numerical Analysis (CBCS-
LOCF)

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V – DSE-1

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting any **two** parts from each question.
3. Marks for each part are indicated on right hand side in brackets.
4. Use of non-programmable Scientific Calculator is allowed.

P.T.O.

1. (a) Explain geometrically the method of false position to determine an approximation to a root of $f(x) = 0$. Applying this method, perform three iterations to determine an approximation to a root of $x^4 - 18x^2 + 45 = 0$ lying in $(3,4)$. (6)

- (b) Perform four iterations of the Bisection method to approximate the root of the equation $x^3 - 3x^2 - 0.09x + 0.27 = 0$ in the interval $(0,1)$. (6)

- (c) Consider the function

$$f(x) = \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$$

What is the limit of f as $x \rightarrow 0$? Further, at what rate does f converge to this limit? (6)

2. (a) Apply Secant method to $x^3 + x^2 - 3x - 3 = 0$ to determine an approximation to a root lying in the interval $(1,2)$. Perform four iterations. (6.5)
- (b) Find the iterative method based on Newton's method for finding $N^{1/3}$, where N is a positive real number. Then perform three iterations of Newton's method to obtain the approximate value of $18^{1/3}$, starting with the initial approximation $x_0 = 2$. (6.5)
- (c) (i) Find the fixed points of the function $g(x) = x(2 - ax)$. Determine the order of convergence of the sequence $p_n = g(p_{n-1})$ toward $x = 1/a$. (3.5)
- (ii) Using Bisection method, determine p_3 , the third approximation to a root of $\cos x - x = 0$ lying in $(0,1)$. (3.0)

3. (a) Starting with initial vector $(x, y, z) = (0, 0, 0)$, perform three iterations of Gauss-Seidel method to solve the following system of equations:

$$6x - 2y + z = 11$$

$$-2x + 7y + 2z = 5$$

$$x + 2y - 5z = -1 \quad (6)$$

- (b) Solve the linear system $AX = b$ using Gaussian elimination with pivoting where

$$A = \begin{bmatrix} 10 & -1 & 2 \\ 1 & 10 & -1 \\ 2 & 3 & 20 \end{bmatrix} \text{ and } b = \begin{bmatrix} 4 \\ 3 \\ 7 \end{bmatrix} \quad (6)$$

- (c) Solve the following system of equations using SOR method with initial approximation as $(x, y, z) = (0, 0, 0)$ and $\omega = 1.25$. Perform three iterations.

$$3x - y + z = -1$$

$$-x + 3y - z = 7$$

$$x - y + 3z = -7 \quad (6)$$

4. (a) Construct the divided difference table for the data

x	0.5	1.5	3.0	5.0	6.5	8.0
F(x)	1.625	5.875	31.0	131.0	282.125	521.0

Hence, find the Newton form of the interpolating polynomial and an approximation to value $f(7)$.

(6.5)

- (b) For the following data, obtain the forward difference polynomial using Gregory-Newton forward difference interpolation. Also, interpolate at $x = 0.25$.

(6.5)

x	0.1	0.2	0.3	0.4	0.5
F(x)	1.40	1.56	1.76	2.00	2.28

(6)

- (c) Obtain the piecewise linear interpolating polynomial for the function $f(x)$ defined by the given data

x	1	2	3	8
F(x)	3	7	21	73

Interpolate at $x=3$ and 7 . (6.5)

5. (a) Find out the approximate value of $f'(2)$ and $f''(2)$ using the forward difference formula for the given data (6)

x	1	2	3	4
f(x)	0.5	1.3	2.1	0.9

(b) Verify that second order central difference approximation formula for the second order derivative provides the exact value of the second derivative, regardless of the value of h , for the function $f(x) = 1$, $f(x) = x$, $f(x) = x^2$ and $f(x) = x^3$, but not for the function $f(x) = x^4$. (6)

(c) Derive the second order forward difference approximation to the second derivative of a function f given by

$$f''(x_0) \approx \frac{f(x_0) - 2f(x_0 + h) + f(x_0 + 2h)}{h^2} \quad (6)$$

6. (a) State the Trapezoidal rule for finding the integral of a continuous function $f(x)$ over a closed $[a, b]$

and use it to approximate the integral $\int_0^1 \frac{1}{1+x} dx$.

Also, find exact value of given integral and the absolute error. (6.5)

(b) Prove that the degree of precision (accuracy) for Simpson's rule is 3. (6.5)

(c) Obtain approximate solution of the given initial value problem (IVP) :

$$\frac{dx}{dt} = x - t; \quad 0 \leq t \leq 2, \quad x(0) = 2,$$

in 4 steps using modified Euler's Method. Also, find the absolute error at each step given that the exact solution of the IVP is $x(t) = e^{-t} + t + 1$.

(6.5)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 6087

I

Unique Paper Code : 32357506

Name of the Paper : Cryptography and Network Security

Name of the Course : CBCS / LOCF B.Sc. (H) Mathematics

Semester : V – DSE-II

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting **two** parts from each question.
3. Parts of the questions to be attempted together.
4. Marks are indicated after each question.
5. Use of a simple calculator is allowed.

P.T.O.

1. (a) Explain the three key objectives Confidentiality, Integrity and Availability of computer security.

(6)

- (b) Define Euler's totient function. Prove that if $n = pq$, where p and q are primes, then $\phi(n) = \phi(p) \phi(q)$. Also, find Euler's totient function $\phi(72)$.

(6)

- (c) Encrypt the following message using Vigenere Cipher system using HUMOR as the keyword.

THE PEPSI IS IN THE REFRIDGERATOR

(6)

2. (a) Given that 5 is a primitive root for the prime 1223, solve the discrete logarithm problem $5^x \equiv 3 \pmod{1223}$. Given that $3^{611} \equiv 1 \pmod{1223}$, determine whether x is even or odd.

(6)

- (b) Illustrate the Feistel cipher structure with the help of a diagram.

(6)

- (c) Describe the Blum-Blum Shub pseudorandom bit generator and use it to generate a sequence of six random bits with $p = 5$, $q = 7$, $x = 2$.

(6)

3. (a) Write a binary representation of the elements of the field $GF(2^4)$ modulo the irreducible polynomial $m(x) = x^4 + x + 1$. Find the multiplicative inverse of $x^2 + 1$ modulo $m(x)$. (6)
- (b) List any six threats to a wireless network with a brief description of each. (6)
- (c) Describe MIME content types with the details of possible subtypes in each content type. (6)
4. (a) Give an overview of the AES key expansion algorithm with a short description of the function g . (6.5)
- (b) Given the entries of the first column of a state matrix in hexadecimal form $\begin{matrix} 1B \\ 23 \\ 45 \\ 86 \end{matrix}$, compute the outcome of the MixColumns transformation on IB. Describe the rationale behind MixColumn Transformation. (6.5)
- (c) Compute the substitution for the byte 10 in AES SubByte transformation. (6.5)

5. (a) Define a cryptographic hash function. Give three applications of cryptographic hash function in cryptography. (6.5)
- (b) Suppose Alice chooses two primes $p = 7$, $q = 11$ and she wants to use the RSA cryptosystem. Can he choose $e = 13$ as his public key? If no, give the reason. If yes, find his private key and encrypt the message $m = 15$. (6.5)
- (c) Describe the key differences between Elliptic Curve Cryptosystem and RSA Algorithm for as public key cryptosystems. (6.5)
6. (a) Describe the SHA-512 round function technique with the help of a diagram. (6.5)
- (b) State ElGamal digital signature scheme and verify with the help of an example. Also, explain existential forgery in this method. (6.5)
- (c) Let $q = 29$, $a = 4$ and $b = 20$, consider Elliptic curve $E_q(a, b): y^2 = x^3 + ax + b \pmod{q}$. Show that $A = (5, 22)$ and $B = (16, 27)$ lie on $E_q(a, b)$. Also, prove that $E_q(a, b)$ is nonsingular and find W , where $W = A + B$. (6.5)

26/17/24

[illegible]

Name of the Paper : Probability Theory and Statistics

Name of the Course : **B.Sc. (H) Mathematics**

Duration : 3 Hours

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each Section.

Section I

- P.T.O.

(b) Given the cdf :

6

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{x+2}{4} & -1 \leq x < 1 \\ 1 & 1 \leq x \end{cases}$$

Find : $P\left(-\frac{1}{2} < X \leq \frac{1}{2}\right)$, $P(X=1)$ and $P(2 < X \leq 3)$.

(c) Let $\{C_n\}$ be an increasing sequence of events. Then prove that :

6

$$\lim_{n \rightarrow \infty} P(C_n) = P\left(\lim_{n \rightarrow \infty} C_n\right) = P\left(\bigcup_{n=1}^{\infty} C_n\right).$$

Section II

2. (a) If

$$f(x) = \begin{cases} xe^{\frac{x^2}{2}}, & x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

(i) Show that $f(x)$ is p.d.f.

(ii) Find its distribution function.

6.5

(b) Let the random variable X have mean μ , standard deviation σ and moment generating function $M(t)$, $-h < t < h$. Show that :

6.5

$$E\left(\frac{X-\mu}{\sigma}\right) = 0, E\left(\frac{X-\mu}{\sigma}\right)^2 = 1$$

and

$$M\left\{\exp\left[t\left(\frac{X-\mu}{\sigma}\right)\right]\right\} = e^{\frac{-\mu t}{\sigma}} M\left(\frac{t}{\sigma}\right), \quad -h\sigma < t < h\sigma.$$

(c) If X and Y are independent Poisson variates such that :

6.5

$$P(X = 1) = P(X = 2) \text{ and } P(Y = 2) = P(Y = 3).$$

Find $\text{Var}(X - 2Y)$.

Section III

3. (a) If a random variable has a uniform density with the parameters α and β , find its distribution function. 6
- (b) A die is tossed until 6 appears. What is the probability that it must be tossed more than 5 times ? 6
- (c) At a certain location on highway I-10, the number of cars exceeding the speed limit by more than 10 miles per hour in half an hour is a random variable having a Exponential distribution with $\lambda = 8.4$. What is the probability of a waiting time of less than 5 minutes between cars exceeding the speed limit by more than 10 miles per hour ? 6

Section IV

4. (a) Suppose that the amount of cosmic radiation in a man has a normal distribution with mean 4.35 and standard deviation 0.59. Find the probability that a person will be exposed to more than 5.2 units of cosmic radiation. 6.5

[Given : $\frac{1}{\sqrt{2\pi}} \int_0^{1.44} e^{-\frac{t^2}{2}} = 0.4251$]

- (b) Let the random variables X and Y have the joint pdf, 6.5

$$f(x, y) = x + y, \quad 0 < x < 1, \quad 0 < y < 1, \quad \text{zero elsewhere.}$$

Determine the conditional mean and variance of Y, given $X = x$, for $0 < x < 1$.

- (c) Let X and Y be two random variables with joint pdf

$$f(x, y) = xe^{-y}, \quad 0 < x < y < \infty, \quad \text{zero elsewhere.}$$

Determine the joint moment generating function $M(t_1, t_2)$ of X and Y. Is

$$M(t_1, t_2) = M(t_1, 0) M(0, t_2) ? \quad 6.5$$

Section V

5. (a) Prove that if μ and σ are the mean and the standard deviation of a random variable X, then for any positive constant k the probability is at least $1 - \frac{1}{k^2}$ that X will take on a value within k standard deviations of the mean, i.e. 6

$$P(|X - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}, \quad \sigma \neq 0.$$

- (b) Let the random variables X and Y have the joint pmf :

6

$$p(x, y) = \frac{1}{3}, (x, y) = (0, 0), (1, 1), (2, 2), \text{ zero elsewhere.}$$

Compute the correlation coefficient of X and Y.

- (c) Let the random variables X and Y have the joint pdf,

6

$$f(x, y) = 2, 0 < x < y < 1, \text{ zero elsewhere.}$$

- (i) Determine the marginal pdfs $f_X(x)$ and $f_Y(y)$.

- (ii) Also find the conditional mean $E(X|y)$ of X, given $Y = y$, for $0 < y < 1$.

Section VI

6. (a) Suppose we know that the number of items produced in a factory during a week is a random variable with mean 500.

6.5

- (i) What can be said about the probability that this week's production will be at least 1000 ?

- (ii) If the variance of a week's production is known to equal 100, then what can be said about the probability that this week's production will be between 400 and 600 ?

- (b) State and prove the Central Limit Theorem.

6.5

- (c) The following data gives the aptitude test scores and productivity indices of 10 workers selected at random :

6.5

Aptitude scores (X) :	60	62	65	70	72	48	53	73	65	82
Productivity index (Y) :	68	60	62	80	85	40	52	62	60	81

- (i) Find the equation of the least squares line that approximates the regression of the productivity index on the Aptitude test.
- (ii) Predict the average productivity index of a worker whose test score is 92.